EE215 – FUNDAMENTALS OF ELECTRICAL ENGINEERING

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WEEK 10
SECOND ORDER CIRCUIT RESPONSE

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PARALLEL RLC CIRCUITS: UNDERDAMPED VOLTAGE RESPONSE

- $\alpha^2 - \omega_0^2 < 0$ ➔
- We could use the same approach as in the overdamped case ($\alpha^2 - \omega_0^2 > 0$) and determine $\boldsymbol{v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}}$
- But it is more convenient to rearrange the solution and avoid the complex numbers.
- Euler identity:

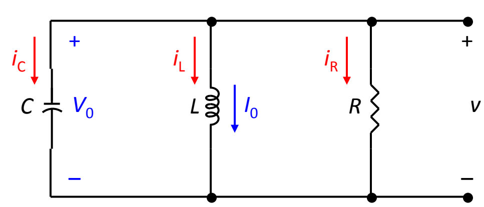
PARALLEL RLC CIRCUITS: UNDERDAMPED VOLTAGE RESPONSE

PARALLEL RLC CIRCUITS: UNDERDAMPED VOLTAGE RESPONSE

- Let
  - Note that B_1 and B_2 are real (i.e., not complex!) because A_1 and A_2 are complex conjugates.
-
- Determine B_1 and B_2 :
-

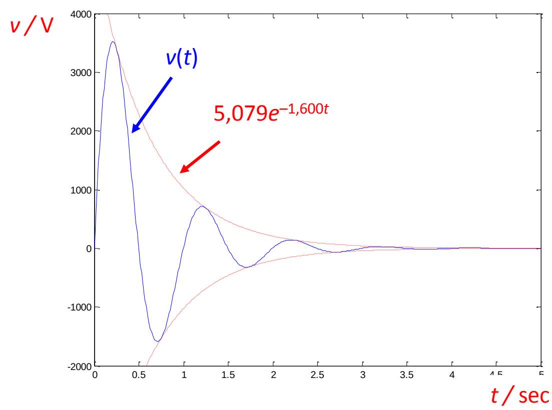
UNDERDAMPED PARALLEL RLC CIRCUIT EXAMPLE (1)

- $R = 2500\Omega$
- $L = 189.3\text{mH}$
- $C = 125\text{nF}$
- $V_0 = 0\text{V}$, $I_0 = -4\text{A}$



UNDERDAMPED PARALLEL RLC CIRCUIT EXAMPLE (2)

- What is $v(t)$, $t \geq 0$?



CHARACTERISTICS OF THE UNDERDAMPED RESPONSE

- $v(t) = e^{-\alpha t} (B_1 \cos \omega_d t + B_2 \sin \omega_d t)$

- Without damping,

- Damping reduces
- Damping reduces

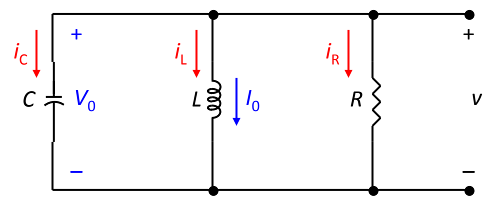
CRITICALLY DAMPED RESPONSE OF THE PARALLEL RLC CIRCUIT

- $\alpha^2 = \omega_0^2$
- Our previous approach does not work in this special case.
  –
  – This equation cannot be the solution for all initial conditions of V_0 and I_0 (there is only one parameter).

CRITICALLY DAMPED RESPONSE OF THE PARALLEL RLC CIRCUIT

- The solution of the critically damped circuit is given by
- We will not derive this formula here.

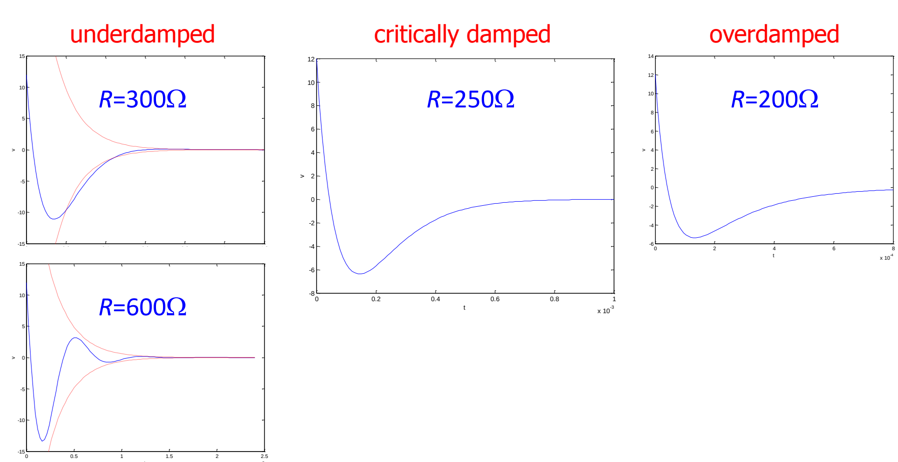
CRITICALLY DAMPED PARALLEL RLC CIRCUIT EXAMPLE (1)



***R* = 250Ω,**
***L* = 50mH,**
***C* = 0.2μF,**
V_0=12V, I_0 = 30mA

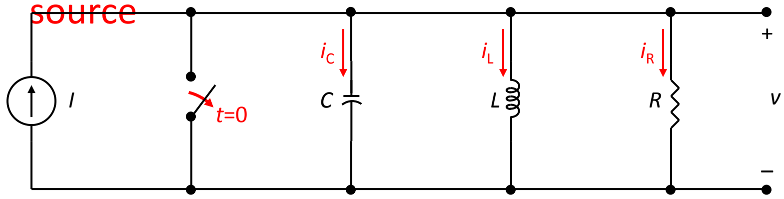
CRITICALLY DAMPED PARALLEL RLC CIRCUIT EXAMPLE (2)

$$v(t) = -270{,}000te^{-10{,}000t} + 12e^{-10{,}000t}$$



STEP RESPONSE OF THE PARALLEL RLC CIRCUIT

- Step response: sudden application of current source



- KLC:
- Rearranging:
- Derivative and divide by C:

STEP RESPONSE OF THE PARALLEL RLC CIRCUIT

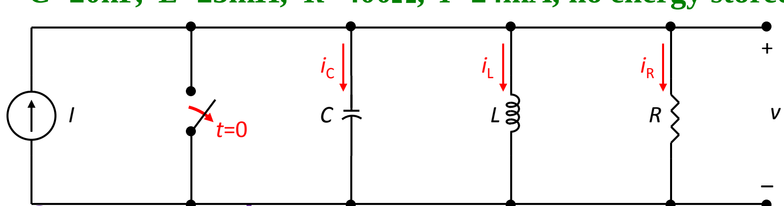
- This analysis leads to the same characteristic equation as in the natural response.
  - ➔ Voltage $v(t)$ has the same solution form.
- As in natural response, we get
  with
- The solution for overdamped, underdamped, and critically damped responses are analogous to the solutions of the natural response.

WHAT ABOUT THE BRANCH CURRENTS?

- $\boldsymbol{i_L}$ is particularly interesting because $\boldsymbol{i_L(\infty) \neq 0}$ in general.

EXAMPLE: STEP RESPONSE OF THE PARALLEL RLC CIRCUIT (1)

- **C=20nF, L=25mH, R=400Ω, I=24mA, no energy stored**



- **Some general comments:**
- At $\boldsymbol{t = 0^-}$, $\boldsymbol{v = 0}$ and $\boldsymbol{i_L = 0}$.
- Inductor
- Capacitor

EXAMPLE: STEP RESPONSE OF THE PARALLEL RLC CIRCUIT (2)

- As the capacitor charges up:
  –
- In steady state:
  –
- The way this steady state is reached depends on the damping of the circuit:
  – Many oscillations
  – Steady rise and fall
  – Very gradual approach

EXAMPLE: STEP RESPONSE OF THE PARALLEL RLC CIRCUIT (3)

- The example above is overdamped:

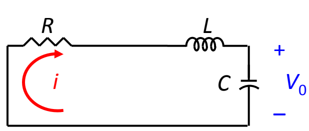
- Roots of characteristic equation:
-
- Current through inductor:

SUMMARY: NATURAL AND STEP RESPONSE OF PARALLEL RLC CIRCUIT

Damping	Equation	Coefficient Equations
Over	$v(t) = v_\infty + A_1' e^{s_1 t} + A_2' e^{s_2 t}$	$v(0) = v_\infty + A_1' + A_2'$ $dv/dt(0) = A_1' s_1 + A_2' s_2$
Under	$v(t) = v_\infty + e^{-\alpha t}\left(B_1' \cos \omega_d t + B_2' \sin \omega_d t\right)$	$v(0) = v_\infty + B_1'$ $dv/dt(0) = -\alpha B_1' + \omega_d B_2'$
Critical	$v(t) = v_\infty + \left(D_1' t + D_2'\right) e^{-\alpha t}$	$v(0) = v_\infty + D_2'$ $dv/dt(0) = D_1' - \alpha D_2'$

Note: For natural response, $v_\infty = 0$ and $A_1' \rightarrow A_1$, etc.

NATURAL AND STEP RESPONSE OF THE SERIES RLC CIRCUIT



- KVL:
- Differentiate:
- Rearrange:
- Characteristic equation:

SOLUTIONS TO THE CHARACTERISTIC EQUATION

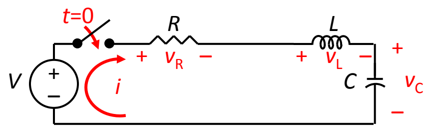
- Neper frequency:
- Resonant radian frequency:
- The solution for the serial RLC circuit has the same form as for the parallel RLC circuit.

SOLUTIONS TO THE CHARACTERISTIC EQUATION

- Overdamped $\alpha^2 > \omega_0^2$: $i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$
- Underdamped $\alpha^2 < \omega_0^2$: $i(t) = e^{-\alpha t}\left(B_1 \cos \omega_d t + B_2 \sin \omega_d t\right)$
- Critically damped $\alpha^2 = \omega_0^2$: $i(t) = D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}$

	Parallel RLC	**Serial RLC**
Char. eq.	involves $\boldsymbol{v}$	involves $\boldsymbol{i}$
α	$\frac{1}{2RC}$	$\frac{R}{2L}$
ω_0	$\frac{1}{\sqrt{LC}}$	$\frac{1}{\sqrt{LC}}$
damping	compare α^2 and ω_0^2	compare α^2 and ω_0^2

STEP RESPONSE OF THE SERIES RLC CIRCUIT



- Note: characteristic equation remains the same.
- KVL:

- This equation, again, has the same form as the condition on $\boldsymbol{i}_L$ in the parallel RLC circuit.